APPLICATION OF LEAD FIELD THEORY AND COMPUTERIZED THORAX MODELING FOR THE ECG INVERSE PROBLEM

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Abstract – **The ECG inverse problem is a widely studied area, and several different approaches have been used to solve it. The present study introduces the reciprocally calculated lead field concept for solving the ECG inverse problem. The lead field approach based on the reciprocity theorem provides a procedure to calculate the computationally heavy forward problem by a single solution for each ECG lead. In this study, one anatomically detailed 3D FDM model of the human thorax as a volume conductor was employed for forward and inverse estimation of ECG potentials and cardiac sources, respectively. Several equivalent dipole sources were set into the cardiac muscle and the surface potential distributions applying 12, 24, 32, 64, and 120-lead ECG electrode configurations were computed. The inverse problem was solved in order to localize the dipoles based on the information obtained from the simulated ECG recordings and the characteristics of the volume conductor. The dipole localization errors ranged from 2 to 5 mm depending on the number of electrodes. Thus, the lead field method appears to be applicable for the solution of the ECG inverse problem.**
Keywords – **Lead field concept, ECG, inverse problem, thorax modeling**

I. INTRODUCTION

The measurement of the electrocardiogram (ECG) provides non-invasively obtained data for localizing the electric sources generating potentials on the surface of the thorax. During a past few decades, the ECG inverse problem has been widely studied in order to establish a way to determine the equivalent source configurations within the body based on this surface potential data. The lead field concept provides one possible approach for linking the information available for the nature of the source generating the currents and the properties of the thorax as a volume conductor [1]. This field provides the relationship between the equivalent dipole modeling the local electric activity of the heart and the body surface potentials measured. Thus the reciprocally calculated lead field can be used for forward simulation of electric potentials generated by the known sources or for the estimation of the inverse solution of the sources generating the measured potentials.

Increased computing capacity has allowed the construction of anatomically detailed, inhomogeneous thorax models [3], however, iterative estimation of a dipole location may take large number of solutions. An important feature of the lead field concept is that number of solutions is equal to the number of the electrodes on the surface of the thorax. It is obvious that the larger the number of electrodes, the more accurate information can be gained regarding the body surface potential distribution [5]. However, it has been shown previously that 32 leads provide a fairly good approximation [6].

The inverse problem is ill-posed by its nature, and thus several approaches have been used to try to solve it. The general scheme to solve the ECG inverse problem, i.e. to localize dipolar sources, has been to assume one source configuration, to calculate the forward solution, and then to compare the calculated potentials with the measured potentials until the optimal fit has been found. Inverse algorithms used for single dipole localization have been generally based on least squares approach or probabilistic methods.

Due to the non-uniqueness of the inverse problem, physiological constraints must be imposed. There may exist numerous possible source locations and directions, and thus the solution of the forward problem can be laborious and time-consuming. Therefore, the development of a reliable and efficient method for both forward calculations and inverse algorithms is important.

The main objective of this study is to demonstrate the usability of reciprocally calculated lead fields for the ECG inverse solution using an anatomically detailed, inhomogeneous volume conductor model of the human thorax. The basic methodological background of the lead field concept is introduced, and the results of simulations performed with one computerized thorax model are represented. Thus, performance of the lead field concept for ECG source localization is evaluated.

II. METHODOLOGY

A. Human Body as a Volume Conductor

The human body can be considered as a resistive, piecewise homogeneous and linear volume conductor [1]. The governing equation of the electrical properties of the body as a volume conductor may thus be written as the Poisson’s equation.

In a finite difference method (FDM) modeling approach applied in this study, the Poisson’ equation is approximated by dividing the volume into a 3-D resistor network that reflects the human body both geometrically and as a conductor. The structures are represented by a 3-D grid of discrete points called nodes, and a network of resistors is placed between these nodes. Resistor values depend on conductivity of the tissue type and the size of the element between the node points.

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The fundamental linear difference equations are set up by applying Ohm's and Kirchoff's laws for each node. The resulting set of linear equations is solved based on the iterative methods. The result of this iterative process is the potential distribution within the volume conductor model due to specific source configurations.

B. Source Models and Their Application for Forward Solutions

Equivalent bioelectric sources are used when solving the *forward problem*, where the solution provides the potential or current distribution on the model surface arising from the known sources of bioelectric origin. Examples of this type of sources are potential distributions due to a dipole or double layer sources in cardiac muscle or within the brain. Externally applied sources require the use of a mixed boundary condition known as Neumann and Dirichlet condition. The problem in which the field and the conductor are known but the source is known is called the *inverse problem* [1]. The inverse problem in ECG is ill-posed in nature and has thus no unique solution. This deficiency can be overcome by setting various constraints to the problem set-up to find a physically meaningful solution. The lead field theory offers one possible approach to the solution of this inverse problem.

C. Application of Reciprocity Theorem and Lead Field Concept for the ECG Inverse Solution

The lead vector and lead field concept were originally developed for describing the sensitivity of bioelectric measurements, but their principles are universal in the sense that they apply to any linear system [1]. The lead vector concept explains the relationship between electromotive source and the measure voltage. For simplification, suppose that a single dipole source vector of magnitude $\overline{p}$ is at fixed location in a linear, resistive volume conductor of arbitrary shape and inhomogeneous conductivity. The measured lead potential Φ_p corresponding to $\overline{p}$ is affected by the proportionality coefficient vector $\overline{c}$ representing three orthogonal components. This transfer vector is dependent on the location of the measurement lead, the source dipole location, and the shape and conductivity distribution of the volume conductor. The linearity assumption ensures that the principle of superposition holds, and the lead voltage V_{ij} can be expressed as the scalar product of the transfer vectors between the measurement locations (e.g. i and j) and the dipole source as

$$V_{ij} = \Phi_i - \Phi_j = \overline{c}_i \cdot \overline{p} - \overline{c}_j \cdot \overline{p} = \overline{c}_{ij} \cdot \overline{p} \tag{3}$$

where $\overline{c}_{ij}$ is the three-dimensional transfer coefficient describing the sensitivity of the bioelectric measurement at a specific location, i.e. the lead vector.

The concept of lead field is a straightforward extension of the lead vector concept, and it is generally applied in connection with the reciprocity theorem. Basically, the lead field can be understood as a continuous vector field comprising of single lead vectors mapped as a function of the source location throughout the volume conductor [1].

Normally, voltage measurement is made on the surface of the volume conductor, and the measured signal in the lead arises from all the sources in the conductor according to (3) at each location. The reciprocity theorem originally introduced by Helmholz in 1853 states that electric field E inside the volume conductor, e.g. the thorax, generated by a reciprocal unit current I_r applied to the surface electrodes expresses how the same electrodes record potentials caused by dipole sources at any location within the volume conductor [1]. Thus, the associated current density field $\overline{J}$ has exactly the same form as the lead field. If individual current dipoles are characterized by current dipole moment per volume $\overline{J}^i$, noting that $J = \sigma E$, the expression for the lead potential becomes

$$V_{LE} = \int_v \overline{c} \cdot \overline{p} \quad dv = \int_v \frac{1}{\sigma} \overline{J}_{LE} \cdot \overline{J}^i \ dv \tag{4}$$

where $\overline{J}_{LE}$ denotes the lead field and σ is the electrical conductivity tensor unique for each location and direction. Thus, the lead field fully takes into account the effect of volume conductor boundary and internal inhomogeneities, i.e. the conductivity distribution.

D. Material – Constructing the FDM Model

In this study, a computerized model of the human thorax as a volume conductor was constructed based on the finite difference method (FDM). One magnetic resonance image set comprising of 70 transverse slices presenting the anatomy during diastole was segmented using the semiautomatic IARD method [7] to construct an anatomically detailed model. Altogether 26 distinct tissue types including e.g. intracavitary blood, pericardium, major vessels, lungs, subcutaneous muscle and fat, bony structures, and some internal organs (liver, spleen) were determined from the MR images [8]. The segmentation procedure assigns a tissue code for every voxel in the original image and this data is as such applicable for FDM calculations. The conductivity values for the different tissues were adopted from the literature [9].

E. Calculation of the Body Surface Potentials

The simulated potential data was generated by inserting an equivalent dipole source into the constructed volume conductor model and calculating the body surface potential distribution with the FDM solver [9]. Four single dipoles at different locations of the cardiac muscle were considered. Each of these dipoles was simulated in x, y, and z directions according the rectangular grid used for

modeling. The locations of the dipoles were as follows: Dipole 1 was located in the septal area in the middle of the heart. Dipole 2 situated in the septal area but closer to the apex than dipole 1. Dipole 3 was applied in the vicinity of the lowest part of the apex, and dipole 4 located laterally in the left ventricle wall.

Thus, in all, 12 cases were considered as simulated potential sets all scaled to correspond the potentials due to a 1mA current source. Finally, the simulated potential data was picked out at the predefined ECG electrode locations determined by each electrode configuration. For testing the inverse algorithm maps with added measurement noise were obtained with 10, 20 and 30% RMS noise.

F. Calculation of the Lead Fields

The same thorax model was employed to calculate the lead fields. A current source was generated between two surface electrodes and a gradient was calculated from the resulting potentials at the nodes of the volume conductor model [9].

A number of closest nodes corresponding to the electrode location according to electrode configuration was set as sourcea. For the calculation of the lead field $\overline{J}_{LE}$ the reciprocal source current was set to 1 A by scaling the potential field by the applied current. The lead field procedure creates the three-dimensional lead vector field for each node of the volume conductor model by a single energization of the measurement lead.

The analyzed ECG lead arrangements included the standard 12-lead system, a modified 24-lead, Lux 32-lead full body, Montreal 64-lead, and Brussels 120-lead electrode configurations. Thus, the forward calculation was repeated 120 times to obtain the lead fields of all ECG channels needed. However, the 120 calculations provided the lead field in over 8000 possible source nodes in the heart. Further, an appropriate number and location of channels were selected among these leads for each analyzed lead configuration.

G. Source Localization Algorithm and Measure of the Accuracy for Dipole Localization

A single equivalent current dipole was assumed as the source model. The inverse solutions were calculated using the maximum likelihood method to estimate the location of the dipoles with the inverse localization program originally developed in our laboratory for EEG studies [10].

The performance of the source localization algorithm was estimated by the error of the location in comparison with the known dipole source parameters

The dipole magnitudes and moments were not compared. In the lead field approach an actual point dipole is used while in the simulated dipoles two nodes with finite grid distance formed the source and the sink of the dipole. Thus, the magnitude values are not directly comparable.

III. RESULTS

A. Forward Calculations

The simulated surface potentials due to dipole sources were obtained from the potential data at the nodes closest to the electrode locations of analyzed lead configurations. The calculations for simulated potential data required 31271 to 63337 iterations to terminate. The average was 53302 iterations.

The lead fields were calculated with an iterative FDM solver for all the nodes in the volume conductor model.

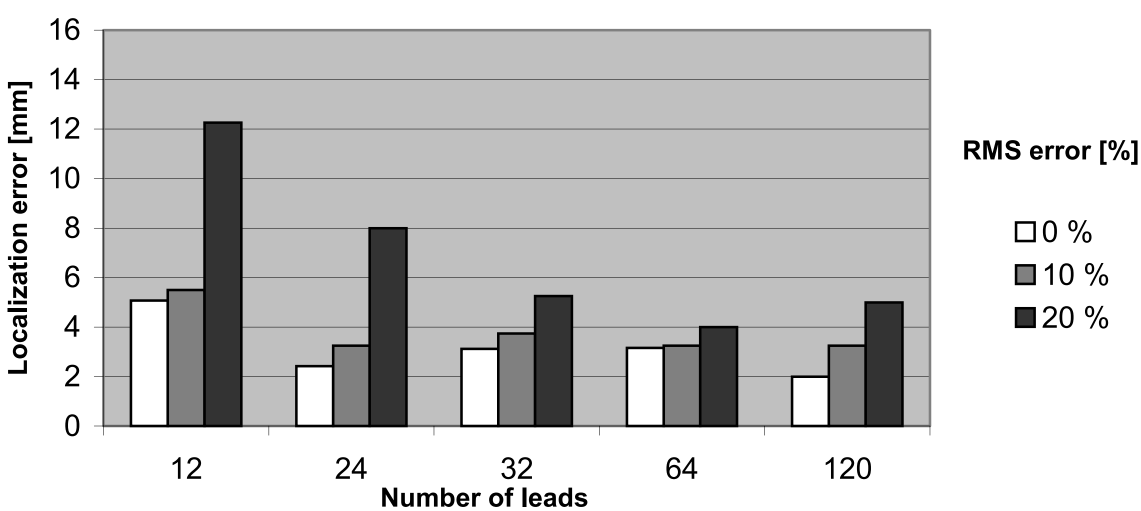

Table 1. Average dipole localization error in millimeters for different electrode configurations considered.

Each lead was considered as bipolar lead, the reference electrode being left leg.

B. Inverse Solution

The inverse solution using the simulated potential data and the calculated lead fields was obtained for all 12 dipoles. The solutions were restricted to the heart area, and thus the solution was to be found among 8219 nodes representing the cardiac muscle.

The average values are calculated to include all four dipoles in all three orientations with no noise and 10%, 20% and 30% RMS noise. These error values considered are summarized in Table. 1. In noiseless case the dependency of the localization accuracy on the electrode configuration was assessed and found to lie between 2 and 5 millimeters. With added noise the accuracy of the measurement systems with low number of electrodes decreased.

IV. DISCUSSION

In this study the lead field approach based on the reciprocity theorem is used to solve the ECG inverse problem. A computerized human thorax model as a volume conductor was constructed, and several single dipole sources were inserted into the cardiac muscle to simulate the electric sources generating the body surface potential distribution. The inverse problem was solved using this surface potential data and the information regarding the properties of the thorax obtained from the lead fields.

The results obtained with the simulations indicate that the applied lead field procedure provides a reliable and relatively method for solving the ECG inverse problem. The dipole localization error produced by the procedure lies well within the range of modeling accuracy (transversally 2 mm, sagittally 5 mm).

The effect of electrode configuration was assessed by comparing the source localization accuracy produced by 5 different electrode configurations. The localization error in case of using the standard 12-lead ECG is slightly higher than when increasing the electrode density. However, in noiseless case the accuracy of 12-lead system was surprisingly good. The model resolution was the limiting factor with large number of electrodes. With added noise the performance of the 12-lead ECG was poor. With 32 or more electrodes the localization accuracy remained good. This is in accordance with previous results obtained by e.g. Lux [6]. Here the localization was based on one dipole model. If more complex sources are to be needed, more leads may be required.

V. CONCLUSION

In this study we have introduced the application of the lead field approach to the ECG inverse problem. The basic theoretical aspects regarding the lead field theory have been introduced and results based on simulations conducted with a computerized human thorax model have been presented. Our results denote that the lead field approach presents a powerful tool with both high localization accuracy and moderate computational load for calculating ECG inverse solutions.

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